Mathematics-Specific Communication

Goals and Objectives

The mathematics department wants students to know that clear expression plays an essential role both in communicating understanding and in refining understanding. Upon encountering unfamiliar mathematics, students should be able to articulate the questions addressed by that mathematics and identify the techniques used to address those questions. Students should be able to master those techniques sufficiently well that they can use them in writing rigorous proofs and/or in constructing reliable algorithms for solving problems. Upon encountering a potential application of mathematics, students should be able to identify the mathematics that can be used to address the application and to analyze that mathematics in the ways described above. Student presentations of mathematics should be tailored to the background and interests of the audience.

In short the department wants students to understand and use communication as the part of discovery that is driven by attention to the perspectives of others. This desire applies equally to written, spoken, and visual communication, each of which takes more than one form. Written communication is unequaled as a way to explore, clarify, and communicate subtle relationships among ideas. It was asynchronous long before the word was fashionable. Written communication of mathematics includes the writing of proofs, mathematical exposition, and computer programs. Spoken communication can be either formal or informal. A strength of spoken communication is the flexibility with which it raises and elicits questions. Visual communication takes both schematic and geometric forms. The former relies on the use of diagrams and similar methods to express logically simple relationships, while the latter relies on the geometric insight that is at the heart of many mathematical questions and insights.

Curriculum

The mathematics department’s attention to student intellectual growth predates the imposition of writing intensive requirements and was not altered by those requirements. For that reason the department’s desired integrated, sequenced approach to communication closely matches the approach currently in place. Unless explicitly noted, the curriculum described below is both the desired one and the current one. It follows that our plans will require no changes in the courses taken by students; checksheets will remain unchanged, with the exception of the removal of references to WI and the inclusion of references to the new communication requirement. Towards the end of this document we describe the steps we are taking to enhance students’ communication-related experiences.

The mathematics department offers four BS degree options: traditional, applied and computational (ACM), applied discrete (ADM), and education (MAED). The discussion below applies to all four options except where differences are noted.
Math 1205-1206. First-year students learn that written answers must include organized logically complete reasoning. Applications problems and some purely mathematical problems require students to break problems into subordinate problems. Class discussions, faculty office hours, and work at the Emporium give students opportunities to use informal spoken communication. Much of the content depends on geometric insight in two dimensions.

Math 2214, 2224. Second-year students increase their mastery of the skills learned in Math 1205-1206 through exposure to more sophisticated problems. For example a single-variable optimization problem of the type encountered in Math 1205 is a subordinate problem in a two-variable optimization problem of the type encountered in Math 2224. An application problem in Math 2214 can involve creating the differential equation that models a physical situation, finding the general solution of the differential equation, finding the specific solution determined by given initial conditions, and using that specific solution to answer questions about the physical situation. Geometric techniques encountered in Math 2224 involve the use of graphs in three dimensions to visualize the behavior of functions of two variables, and the use of level sets to visualize the behavior of functions whose graphs are difficult to visualize. Math 2214 introduces elementary ways to think about geometry in infinite-dimensional spaces of functions.

CS 1044. Students learn to program.

Math 3034. Through extensive, thoroughly criticized writing, students begin to learn to write mathematical proofs.

Math 3124, 3224. (traditional, ADM, and MAED) Third-year students expand their ability to write and use mathematical proofs by writing proofs while learning challenging new mathematics. Students experience the process through which intuition is translated into a rigorous proof that informs deeper intuition. Most students engage in substantial amounts of informal spoken communication with their teachers and fellow students. Some sections of Math 3124 rely on geometric visualization of symmetry groups.

Math 3224 and 12 hours in a single applied discipline. (ACM) The comments for Math 3224 are as above. Math department academic advisors approve the 12 hours in a single applied discipline for ACM students on an individual basis. One goal of this requirement is that students become familiar with the style of discourse in the applied discipline.

4000-level sequence. Fourth-year traditional, ACM, and ADM students take 6 hours from a sequence or cluster appearing on an approved list. (The ACM requirement uses a more restrictive list.) Students in these courses write proofs, algorithms, and/or programs while learning challenging mathematics in the depth permitted by a year-long course that has 3000-level prerequisites. Examples of the work encountered in these courses are as follows. Each of Math 4225, 4226, and 4124 involves forty pages of written proofs, with some of the individual proofs one page or more in length. Math 4425, 4426, 4445, and 4446 involve that much writing, but some of the writing can take the form of programs,
and these courses sometimes substitute substantial projects for some of the weekly assignments. The same is true of Math 4164. All of these courses involve most of their students in substantial amounts of informal spoken communication. Some of the projects include formal spoken presentations. Most of these courses involve expressing rigorously ideas based on the extension of geometric insight from three dimensions to high finite dimensions or infinite dimensions.

**4000-level (MAED)** Fourth-year MAED students take Math 4044 and Math 4654. The former involves the writing of anywhere from 10 to 25 pages of papers on the history of mathematics. The versions of the course that require fewer pages of history papers require frequent written solutions of mathematics problems of historical significance. Students in Math 4654 write over 20 pages of papers on topics in math education research.

An ideal MAED fourth-year communication requirement would include supervised presentation of mathematics to students in pre-college classrooms. MAED students currently get this experience as part of their teacher aide and student teaching classes. Due to decisions that the mathematics department has been unable to influence, these classes will soon be unavailable to undergraduates. MAED students will have to continue into a MS program to complete their preparation for effective communication in their field.

**Summary of Communication Requirements**

**Traditional:** Math 1205-1206, Math 2214, 2224, CS 1044, Math 3034, Math 3124, 3224, approved 6-hour cluster or sequence in 4000-level mathematics.

**ADM:** Math 1205-1206, Math 2214, 2224, CS 1044, Math 3034, Math 3124, 3224, approved 6-hour cluster or sequence in 4000-level mathematics.

**ACM:** Math 1205-1206, Math 2214, 2224, CS 1044, Math 3034, Math 3224, approved 12-hours in an applied discipline, approved 6-hour cluster or sequence in 4000-level mathematics.

**MAED:** Math 1205-1206, Math 2214, 2224, CS 1044, Math 3034, Math 3124, 3224, Math 4044, 4654.

**Dreams**

An ideal fourth-year experience for a strong math major would include a research project resulting in a substantial written paper and a seminar presentation. Currently roughly seven math majors per year do such research and write such papers, with fewer than seven making a seminar presentation as well as writing a paper. The department encourages such papers and presentations by awarding a cash prize, the Layman Prize, to the student whose paper and presentation, considered together, are judged to be the best. The most recent annual report sets a goal of twenty undergraduate research projects per
year. The department has applied for a $1.1 million grant to put in place an integrated, sequenced approach to preparing students for research projects.

**Assessment**

Because the department regards communication and learning as inseparable, most of the assessment of communication will follow the assessment model established in the most recent outcomes assessment. This assessment is based largely on statistical measures of student success, but also includes employment information and survey data. The department will add to the surveys a question or questions about communication skills.

One aspect of the department’s assessment stands out for its relevance to communication. In senior oral examinations three faculty members meet with small groups of students to ask those students questions about mathematics they should recall even a few years after taking the relevant courses. The students answer as a group, while the faculty members observe the discussion that occurs as the students prepare the group answer. This process reveals not only student recollection of content, but also student thought processes, as well as student mastery of informal spoken communication skills.

The department keeps copies of all undergraduate research papers written by its students. The number of such papers will appear in the department’s assessment reports. An analysis of the quality and content of these papers, as well as of the quality of the associated seminar presentations, will become part of the department’s assessment process.